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Some aspects on generalization of struve transformation

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Abstract : This paper deals with some aspects of Struve transformation which has been extended to a class of generalized function.

Keywords : Struve Transformation, Generalized Function, Testing Function.

INTRODUCTION

The classical Struve Transformation of a function *f* is defined by

$$\left(H_\nu f\right)(x) = \int_0^\infty (xt)^{1/2} H_\nu(xt) f(t)\,dt \quad 1.1$$

Where $H_\nu(z)$ is the Struve function of order *v* given by [*of Love(2) Rooney (1)*] it in detail. The aim of present work is to extend the Struve transform to a class of generalized function and establish its inversion.

DEFINITION

The Struve function $H_\nu(z)$ of order *v* is defined by

$$\Gamma\left(\nu+\frac{1}{2}\right)H_\nu(z) = \frac{2\left(\frac{z}{2}\right)^\nu}{\sqrt{\pi}} \int_0^{\pi/2} \sin(z\cos\phi)\sin^{2\nu}\phi\,d\phi \quad 2.1$$

Or, $$H_\nu(z) = \sum_{m=0}^\infty \frac{(-1)^m \left(\frac{z}{2}\right)^{\nu+2m+1}}{\Gamma\left(m+\frac{3}{2}\right)\left(\nu+m+\frac{3}{2}\right)};$$

m = 1, 2, 3……….. for all value of *v* 2.2

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2. The testing function Space $H_\alpha(I)$ and its dual $H_\alpha'(I)$

For a fixed positive number *á* and an interval $I(0,\infty)$

$H_\alpha(I)$ to be set of all those complex valued smooth function $\phi(t)$ defined on *I* if

$$\gamma_k^\alpha(\phi) = \underset{0<t<\infty}{Sup} \left| e^{-\alpha t}\left(t\frac{d}{dt}\right)^k \varphi(t)\right| < \infty \quad 3.1$$

We see that for each *k* = 0, 1, 2 ……………

$\gamma_k^\alpha(\phi)$ is semi norm on whileis a normequipped with the topology generated by 0, 1, 2 ……become countably multinomed space if $\{\phi_n\}_{n=1}^\infty$ converges to $\phi \in H_\alpha(I)$ then $\{\phi_n\}_{n=1}^\infty$ is a Cauchy sequence in $H_\alpha(I)$

If $\{\phi_n\}_{n=1}^\infty$ be a sequence of functions $H_\alpha(I)$ in converging to zero when $n \to \infty$ then the non – negative integer $r, \{D^r\phi_n\}$ converges to zero uniformly on every compact subset of $(0,\infty)$ as $n \to \infty$. We define $H_\alpha'(I)$ to be the set of all those complex valued smooth function $\phi(t)$ defined on *I* if

$$\underset{0<t<\infty}{Sup} \left| e^{-\alpha t} t^r \frac{d^r}{dt^r}[\phi(t)] \right| < \infty \qquad 3.2$$

$H_\alpha'(I) = \{\phi : \phi \in c^\infty(0,\infty)$ and $\underset{0<t<\infty}{Sup} \left| e^{-\alpha t} t^r \frac{d^r}{dt^r}[\phi(t)] \right| < \infty$

r = 1, 2, …………} 3.3

where c is a constant not depend on n

$H_\alpha'(I)$ are defined in a way similar to these defined in $H_\alpha(I)$. The space $H_\alpha'(I)$ is a complete countably multinormed space and hence is a testing function space.

3. Lemma

For $\alpha > 0, \mathrm{Re}\left(\nu + \frac{1}{2}\right) > 0, t, x > 0$ then,

(i) $(xt)^{1/2} H_\nu(x,t) \in H_\alpha(I)$ 4.1

(ii) $D_x^m\left[(xt)^{1/2} H_\nu(x,t)\right] \in H_\alpha(I)$ 4.2

Proof : By the differential prop. of struve function we have

$$\left(t\frac{d}{dt}\right)^k \left(\sqrt{z} H_\nu(z)\right) = \sum_{j=0}^{k} a_j(\nu) z^{1/2+j} H_{\nu-j}(z)$$

where is a polynomial in ν.

Hence, we have

$$P_r^\alpha[h(xt)] = \underset{0<t<\infty}{Sup} \left| e^{-\alpha t}\left(t\frac{d}{dt}\right)^r [h(xt)] \right| \qquad 4.3$$

$\sim \left| e^{-\alpha t}(xt)^{\nu+\frac{1}{2}} \right| \to 0$ as $t \to 0$ if $\mathrm{Re}\left(\nu + \frac{1}{2}\right) > 0$

This show that $h(x,t) \in H_\alpha(I)$

Under the same conditions

$D_x^m\left[(xt)^{1/2} H_\nu(xt)\right] \in H_\alpha(I)$

4. The generalized transform. For $f \in H_\alpha'(I)$, the generalized Struve transform is defined by

$$s[f] = F(x) = < f(t), \sqrt{(xt)} H_\nu(xt) > \qquad 5.1$$

where x is a non – zero real number and t> 0. From Lemma, we know that for fixed x> 0.

$\sqrt{(xt)} H_\nu(xt) \in H_\alpha(I),$

where ν> -½, $\alpha > 0$. The relation (5.1) is meaningful.

Theorem 5.1

Let a) $f \in H_\alpha'(I)$

and b) $F(x) = < f(t), h(xt) > 0 < x < \infty$

Then $F(x) \in H_\alpha(I)$ for á> 0, $\mathrm{Re}\left(\nu + \frac{1}{2}\right) > 0$

Proof :

From Lemma, to show that $F(x) \in H_\alpha(I)$, it is sufficient to show that

(i) $F(x) \in C^\infty(0,\infty), 0 < x < \infty$ and

(ii) $Sup\left| e^{-\alpha x} x^n \frac{d^n F(x)}{dx^n} \right| < \infty, \alpha > 0.$

Now (i) is obvious from Theorem 4 [N. K. Agrawal and Vijay Kumar [5]

Also,

$$\frac{d^n F(x)}{dx^n} = \frac{\partial^n}{\partial x^n} < f(t), h(xt) >$$

$$= < f(t), \frac{\partial^n h(xt)}{\partial x^n} >$$

From boundedness property of generalized function, we have

$$\frac{d^n F(x)}{dx^n} \le C \max_{0 \le 1 \le q} \gamma_1^\alpha \frac{\partial^n h(xt)}{\partial x^n} \qquad 5.2$$

where C and q are constant depending upon *'f'*. Hence.

$$\left| e^{-\alpha x} x^n \frac{d^n F(x)}{dx^n} \right|$$

$$\le C \max_{0 \le 1 \le q} \underset{0<t<\infty}{Sup} \left| e^{-\alpha x} x^n e^{-\alpha x} t^1 \frac{\partial^1}{\partial t^1} \frac{\left(d^n h(xt)\right)}{dx^n} \right|$$

$$\le C \max_{0 \le 1 \le q} \underset{0<t<\infty}{Sup} \left| e^{-\alpha(x+t)} x^n t^1 \frac{\partial^1}{\partial t^1} \left(\sum_{j=0}^{n} b_j(\nu) x^{j-n} t^j \sqrt{xt} H_{\nu-j}(xt) \right) \right|$$

$$\le C \max_{0 \le 1 \le q} \underset{0<t<\infty}{Sup} \left| e^{-\alpha(x+t)} x^n t^1 \; x^{l-n} \sum_{j=0}^{n} \sum_{p=0}^{l} b_j(\nu) A_{(\nu,j,p)}(xt)^{j-l+p+1/2} H_{\nu-j-p}(xt) \right|$$

$$\le C \max_{0 \le 1 \le q} \underset{0<t<\infty}{Sup} \left| e^{-\alpha(x+t)} \sum\sum \{ b_j(\nu) A_{(\nu,j,p)}(xt)^{j+p+1/2} H_{\nu-j-p}(xt) \} \right|$$

$< \infty$ for $\operatorname{Re}\left(v+\frac{1}{2}\right)>0$, *á*> 0.

Hence (ii) is proved.

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